Exercises 9–5

1. Classify each as independent or dependent samples.
   a. Heights of identical twins
   b. Test scores of the same students in English and psychology
   c. The effectiveness of two different brands of aspirin
   d. Effects of a drug on reaction time, measured by a before and an after test
   e. The effectiveness of two different diets on two different groups of individuals

For Exercises 2 through 10, perform each of these steps. Assume that all variables are normally or approximately normally distributed.
   a. State the hypotheses and identify the claim.
   b. Find the critical value(s).
   c. Compute the test value.
   d. Make the decision.
   e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

2. Sick Days A program for reducing the number of days missed by food handlers in a certain restaurant chain was conducted. The owners hypothesized that after the program the workers would miss fewer days of work due to illness. The table shows the number of days 10 workers missed per month before and after completing the program. Is there enough evidence to support the claim, at $\alpha = 0.05$, that the food handlers missed fewer days after the program?

<table>
<thead>
<tr>
<th>Before</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>7</th>
<th>4</th>
<th>5</th>
<th>3</th>
<th>1</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>After</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

3. Improving Study Habits As an aid for improving students’ study habits, nine students were randomly selected to attend a seminar on the importance of education in life. The table shows the number of hours each student studied per week before and after the seminar. At $\alpha = 0.10$, did attending the seminar increase the number of hours the students studied per week?

<table>
<thead>
<tr>
<th>Before</th>
<th>9</th>
<th>12</th>
<th>6</th>
<th>15</th>
<th>3</th>
<th>18</th>
<th>10</th>
<th>13</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>After</td>
<td>9</td>
<td>17</td>
<td>9</td>
<td>20</td>
<td>2</td>
<td>21</td>
<td>15</td>
<td>22</td>
<td>6</td>
</tr>
</tbody>
</table>

4. Exercise Attitudes A doctor is interested in determining whether a film about exercise will change 10 persons’ attitudes about exercise. The results of his questionnaire are shown. A higher numerical value shows a more favorable attitude toward exercise. Is there enough evidence to support the claim, at $\alpha = 0.05$, that there was a change in attitude? Find the 95% confidence interval for the difference of the two means.

<table>
<thead>
<tr>
<th>Before</th>
<th>12</th>
<th>11</th>
<th>14</th>
<th>9</th>
<th>8</th>
<th>6</th>
<th>8</th>
<th>5</th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>After</td>
<td>13</td>
<td>12</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

5. Sleep Report Students in a statistics class were asked to report the number of hours they slept on weeknights and on weekends. At $\alpha = 0.05$, is there sufficient evidence that there is a difference in the mean number of hours slept?

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours, Sun.–Thurs.</td>
<td>8</td>
<td>5.5</td>
<td>7.5</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Hours, Fri.–Sat.</td>
<td>4</td>
<td>7</td>
<td>10.5</td>
<td>12</td>
<td>11</td>
<td>9</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

6. Legal Costs A sample of municipalities’ legal costs (in thousands of dollars) for two recent consecutive years is as shown. At $\alpha = 0.05$ is there a difference in the costs? Suggest a reason for the difference, if one exists.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>61</td>
<td>26</td>
<td>9</td>
<td>16</td>
<td>61</td>
<td>71</td>
<td>14</td>
<td>86</td>
</tr>
<tr>
<td>Year 2</td>
<td>62</td>
<td>40</td>
<td>10</td>
<td>23</td>
<td>38</td>
<td>118</td>
<td>18</td>
<td>67</td>
</tr>
</tbody>
</table>

Source: Pittsburgh Tribune-Review.

7. Reducing Errors in Grammar A composition teacher wishes to see whether a new grammar program will reduce the number of grammatical errors her students make when writing a two-page essay. The data are shown here. At $\alpha = 0.025$, can it be concluded that the number of errors has been reduced?

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Errors before</td>
<td>12</td>
<td>9</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Errors after</td>
<td>9</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

8. Legal Costs A sample of legal costs (in thousands of dollars) for school districts for two recent consecutive years is shown. At $\alpha = 0.05$ is there a difference in the costs? Suggest a reason for the difference, if one exists.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>108</td>
<td>36</td>
<td>65</td>
<td>108</td>
<td>87</td>
<td>94</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>Year 2</td>
<td>138</td>
<td>28</td>
<td>67</td>
<td>181</td>
<td>97</td>
<td>126</td>
<td>18</td>
<td>67</td>
</tr>
</tbody>
</table>

Source: Pittsburgh Tribune-Review.

9. Pulse Rates of Identical Twins A researcher wanted to compare the pulse rates of identical twins to see whether there was any difference. Eight sets of twins were selected. The rates are given in the table as number of beats per minute. At $\alpha = 0.01$, is there a significant difference in the average pulse rates of twins? Find the
Section 9-5 Testing the Difference Between Two Means: Small Dependent Samples

99% confidence interval for the difference of the two. Use the P-value method.

| Twin A | 87 | 92 | 78 | 83 | 88 | 90 | 84 | 93 |
| Twin B | 83 | 95 | 79 | 83 | 86 | 93 | 80 | 86 |

10. Assessed Land Values A reporter hypothesizes that the average assessed values of land in a large city have changed during a 5-year period. A random sample of wards is selected, and the data (in millions of dollars) are shown. At \( \alpha = 0.05 \), can it be concluded that the average taxable assessed values have changed? Use the P-value method.

<table>
<thead>
<tr>
<th>Ward</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>184</td>
<td>414</td>
<td>22</td>
<td>99</td>
<td>116</td>
<td>49</td>
<td>24</td>
<td>50</td>
<td>282</td>
<td>25</td>
<td>141</td>
<td>45</td>
<td>12</td>
<td>37</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>1999</td>
<td>161</td>
<td>382</td>
<td>22</td>
<td>109</td>
<td>120</td>
<td>52</td>
<td>28</td>
<td>50</td>
<td>287</td>
<td>40</td>
<td>148</td>
<td>56</td>
<td>20</td>
<td>38</td>
<td>9</td>
<td>19</td>
</tr>
</tbody>
</table>

Source: *Pittsburgh Tribune-Review.*

### Extending the Concepts

11. Instead of finding the mean of the differences between \( X_1 \) and \( X_2 \) by subtracting \( X_1 - X_2 \), one can find it by finding the means of \( X_1 \) and \( X_2 \) and then subtracting the means. Show that these two procedures will yield the same results.

### Technology Step by Step

**MINITAB Step by Step**

**Test the Difference Between Two Means: Small Dependent Samples**

For Example 9-12, test the effectiveness of the vitamin regimen. Is there a difference in the strength of the athletes after the treatment?

1. Enter the data into C1 and C2. Name the columns Before and After.
2. Select *Stat>Basic Statistics>Paired T.*
3. Double-click C1 Before for First sample.
4. Double-click C2 After for Second sample. The second sample will be subtracted from the first. The differences are not stored or displayed.
5. Click [Options].
6. Change the Alternative to less than.
7. Click [OK] twice.

![MINITAB Paired T Test and Confidence Intervals](image)
21. \( H_0: \mu_1 - \mu_2 \leq 8 \) (claim) and \( H_1: \mu_1 - \mu_2 > 8 \); C.V. = 1.65; \( z = -0.73 \); do not reject. There is not enough evidence to reject the claim that private school students have exam scores that are at most 8 points higher than those of students in public schools.

**Exercises 9-3**

1. The variance in the numerator should be the larger of the two variances.
2. One degree of freedom is used for the variance associated with the numerator, and one is used for the variance associated with the denominator.
3. a. d.f.N. = 15, d.f.D. = 22; C.V. = 3.36
   b. d.f.N. = 24, d.f.D. = 13; C.V. = 3.59
   c. d.f.N. = 45, d.f.D. = 29; C.V. = 2.03
   d. d.f.N. = 20, d.f.D. = 16; C.V. = 2.28
   e. d.f.N. = 10, d.f.D. = 10; C.V. = 2.98
4. Specific \( P \)-values are in parentheses.
   a. 0.025 < \( P \)-value < 0.05 (0.033)
   b. 0.05 < \( P \)-value < 0.10 (0.072)
   c. \( P \)-value = 0.05
   d. 0.005 < \( P \)-value < 0.01 (0.006)
   e. \( P \)-value = 0.05
   f. \( P \)-value > 0.10 (0.112)
   g. 0.05 < \( P \)-value < 0.10 (0.068)
   h. 0.01 < \( P \)-value < 0.02 (0.015)
5. \( H_0: \sigma_1^2 = \sigma_2^2 \) and \( H_1: \sigma_1^2 \neq \sigma_2^2 \) (claim); C.V. = 2.53; d.f.N. = 14; d.f.D. = 14; \( F = 4.52 \); reject. There is enough evidence to support the claim that there is a difference in the variances.
6. \( H_0: \sigma_1^2 = \sigma_2^2 \) and \( H_1: \sigma_1^2 \neq \sigma_2^2 \) (claim); C.V. = 2.86; d.f.N. = 15; d.f.D. = 15; \( F = 7.85 \); reject. There is enough evidence to support the claim that the variances are different. Since both data sets vary greatly from normality, the results are suspect.
7. \( H_0: \sigma_1^2 = \sigma_2^2 \) and \( H_1: \sigma_1^2 \neq \sigma_2^2 \) (claim); C.V. = 4.99; d.f.N. = 7; d.f.D. = 7; \( F = 1 \); do not reject. There is not enough evidence to support the claim that there is a difference in the variances.
8. \( H_0: \sigma_1^2 = \sigma_2^2 \) and \( H_1: \sigma_1^2 \neq \sigma_2^2 \) (claim); C.V. = 2.27; \( F = 4.52 \); reject. There is enough evidence to support the claim that the standard deviations of the ages are different. One reason is that there are many more people who play the slot machines than people who play roulette. This could possibly account for the larger standard deviation in the ages of the players.
9. \( H_0: \sigma_1^2 = \sigma_2^2 \) and \( H_1: \sigma_1^2 \neq \sigma_2^2 \) (claim); C.V. = 4.03; d.f.N. = 9; d.f.D. = 9; \( F = 1.1026 \); do not reject. There is not enough evidence to support the claim that the variances are not equal.
10. \( H_0: \sigma_1^2 = \sigma_2^2 \) and \( H_1: \sigma_1^2 \neq \sigma_2^2 \) (claim); C.V. = 3.87; d.f.N. = 6; d.f.D. = 7; \( F = 3.18 \); do not reject. There is not enough evidence to reject the claim that the variances of the heights are equal.

21. \( H_0: \mu_1 - \mu_2 \leq 8 \) (claim) and \( H_1: \mu_1 - \mu_2 > 8 \); C.V. = 1.65; \( z = -0.73 \); do not reject. There is not enough evidence to reject the claim that private school students have exam scores that are at most 8 points higher than those of students in public schools.

**Exercises 9-4**

1. \( H_0: \sigma_1^2 = \sigma_2^2 \); C.V. = 4.03; \( F = 1.93 \); do not reject. \( H_0: \mu_1 = \mu_2 \) and \( H_1: \mu_1 \neq \mu_2 \) (claim); C.V. = 2.101; d.f. = 18; \( t = -4.02 \); reject. There is enough evidence to support the claim that there is a significant difference in the values of the homes based on the appraisers’ values.
2. \( H_0: \sigma_1^2 = \sigma_2^2 \); C.V. = 3.05; \( F = 1 \); do not reject. \( H_0: \mu_1 = \mu_2 \) (claim) and \( H_1: \mu_1 \neq \mu_2 \); C.V. = 2.732; \( t = -1.61 \); do not reject. There is not enough evidence to reject the claim that the means are equal.
3. \( H_0: \sigma_1^2 = \sigma_2^2 \); C.V. = 14.94; \( F = 1.41 \); do not reject. \( H_0: \mu_1 = \mu_2 \) and \( H_1: \mu_1 > \mu_2 \) (claim); C.V. = 2.764; d.f. = 10; \( t = 1.45 \); do not reject. There is not enough evidence to support the claim that the average number of family day care homes is greater than the average number of day care centers.
4. \( H_0: \sigma_1^2 = \sigma_2^2 \); C.V. = 4.19; \( F = 1.70 \); do not reject. \( H_0: \mu_1 = \mu_2 \) and \( H_1: \mu_1 \neq \mu_2 \) (claim); C.V. = 2.508; d.f. = 22; \( t = -2.97 \); reject. There is enough evidence to support the claim that there is a difference in the average times of the two groups.
5. \( H_0: \sigma_1^2 = \sigma_2^2 \); C.V. = 1.98; \( F = 1.55 \); do not reject. \( H_0: \mu_1 = \mu_2 \) and \( H_1: \mu_1 > \mu_2 \) (claim); C.V. = 1.282; d.f. = 48; \( t = 11.427 \); reject. There is enough evidence to support the claim that the average cost of a movie ticket in London is greater than the average cost of a movie ticket in New York City. One reason for the difference is the rate of exchange of the money.
6. \( H_0: \sigma_1^2 = \sigma_2^2 \); C.V. = 7.15; \( F = 1.23 \); do not reject. \( H_0: \mu_1 = \mu_2 \) and \( H_1: \mu_1 \neq \mu_2 \) (claim); C.V. = 2.228; d.f. = 10; \( t = 0.119 \); do not reject. There is not enough evidence to support the claim that there is a difference in the variance.
7. \$2626.60 < \mu_1 - \mu_2 < \$11,588.64

**Exercises 9-5**

1. a. Dependent
   b. Dependent
   c. Independent
2. Dependent
3. \( H_0: \mu_D \geq 0 \) and \( H_1: \mu_D < 0 \) (claim); C.V. = -1.397; d.f. = 8; \( t = -2.8 \); reject. There is enough evidence to support the claim that the seminar increased the number of hours students studied.
4. \( H_0: \mu_D = 0 \) and \( H_1: \mu_D \neq 0 \) (claim); C.V. = 2.365; d.f. = 7; \( t = 1.6583 \); do not reject. There is not enough evidence to support the claim that the means are different.
7. $H_0: \mu_d = 0$ and $H_1: \mu_d > 0$ (claim); $C.V. = 2.571$; d.f. = 5; $t = 2.24$; do not reject. There is not enough evidence to support the claim that the errors have been reduced.

9. $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu$ (claim); d.f. = 5; $t = 9.238$; $P = 0.01$. Do not reject the claim.

11. Using the previous problem $\bar{D} = -1.5625$, whereas the mean of the before values is 95.375 and the mean of the after values is 96.9375; hence, $\bar{D} = 95.375 - 96.9375 = -1.5625$.

**Exercises 9-6**

<table>
<thead>
<tr>
<th>a.</th>
<th>$\hat{p} = 0.8$, $q = 0.2$</th>
<th>d.</th>
<th>$\hat{p} = 0.5$, $q = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b.</td>
<td>$\hat{p} = 0.7$, $q = 0.3$</td>
<td>e.</td>
<td>$\hat{p} = 0.4$, $q = 0.6$</td>
</tr>
<tr>
<td>c.</td>
<td>$\hat{p} = 0.6$, $q = 0.4$</td>
<td>f.</td>
<td>$\hat{p} = 0.8$, $q = 0.2$</td>
</tr>
</tbody>
</table>

| 1b. | a. 16 | b. 4 | c. 48 | d. 104 | e. 30 |

3. $\hat{p}_1 = 0.533; \hat{p}_2 = 0.3; \hat{p} = 0.44; \bar{q} = 0.56; H_0: \hat{p}_1 = \hat{p}_2$ and $H_1: \hat{p}_1 \neq \hat{p}_2$ (claim); $C.V. = 2.364; z = 2.364; do not reject. There is enough evidence to support the claim that there is a significant difference in the proportions.

5. $\hat{p}_1 = 0.747; \hat{p}_2 = 0.75; \hat{p} = 0.749; \bar{q} = 0.251; H_0: \hat{p}_1 = \hat{p}_2$ and $H_1: \hat{p}_1 \neq \hat{p}_2$ (claim); $C.V. = 1.96; z = 2.00; do not reject. There is enough evidence to support the claim that the proportions are not equal.

7. $\hat{p}_1 = 0.83; \hat{p}_2 = 0.75; \bar{p} = 0.79; \bar{q} = 0.21; H_0: \hat{p}_1 = \hat{p}_2$ (claim) and $H_1: \hat{p}_1 \neq \hat{p}_2$; $C.V. = 1.96; z = 1.39; do not reject. There is not enough evidence to reject the claim that the proportions are equal.

9. $\hat{p}_1 = 0.55; \bar{p} = 0.45; \bar{q} = 0.53; H_0: \hat{p}_1 = \hat{p}_2$ and $H_1: \hat{p}_1 \neq \hat{p}_2$ (claim); $C.V. = 2.58; z = 1.302; do not reject. There is not enough evidence to support the claim that the proportions are different.

11. $\hat{p}_1 = 0.347; \hat{p}_2 = 0.433; \bar{p} = 0.385; \bar{q} = 0.615; H_0: \hat{p}_1 = \hat{p}_2$ and $H_1: \hat{p}_1 \neq \hat{p}_2$ (claim); $C.V. = 1.98; z = 1.03; do not reject. There is not enough evidence to say that the proportion of dog owners has changed ($-0.252 < \hat{p}_1 - \hat{p}_2 < 0.079$). Yes, the confidence interval contains 0. This is another way to conclude that there is no difference in the proportions.

13. $\hat{p}_1 = 0.25; \hat{p}_2 = 0.31; \bar{p} = 0.286; \bar{q} = 0.714; H_0: \hat{p}_1 = \hat{p}_2$ and $H_1: \hat{p}_1 \neq \hat{p}_2$ (claim); $C.V. = 2.58; z = 1.43; do not reject. There is not enough evidence to support the claim that the proportions are different.

15. $0.077 < \hat{p}_1 - \hat{p}_2 < 0.323$

17. $\hat{p}_1 = 0.43; \hat{p}_2 = 0.58; \bar{p} = 0.50; \bar{q} = 0.495; H_0: \hat{p}_1 = \hat{p}_2$ and $H_1: \hat{p}_1 \neq \hat{p}_2$ (claim); $C.V. = 1.96; z = 0.02; do not reject. There is not enough evidence to support the claim that the proportions are different.

19. $-0.0631 < \hat{p}_1 - \hat{p}_2 < 0.0667$. It does agree with the Almanac statistics stating a difference of $-0.042$ since $-0.042$ is contained in the interval.

**Review Exercises**

1. $H_0: \mu_1 \leq \mu_2$ and $H_1: \mu_1 > \mu_2$ (claim); $C.V. = 2.33; z = 0.59; do not reject. There is not enough evidence to support the claim that single drivers do more pleasure driving than married drivers.

3. $H_0: \sigma_1 = \sigma_2$ and $H_1: \sigma_1 \neq \sigma_2$ (claim); $C.V. = 2.77; \alpha = 0.10; d.f.N. = 23; d.f.D. = 10; F = 10.365; reject. There is enough evidence to support the claim that there is a difference in the standard deviations.

5. $H_0: \sigma_1 \leq \sigma_2$ and $H_1: \sigma_1 > \sigma_2$ (claim); $C.V. = 2.05; d.f.N. = 9; d.f.D. = 9; F = 5.06. The $P$-value for the $F$ test is $0.01 < P-value < 0.025 (0.012); reject since $P-value < 0.05$. There is enough evidence to support the claim that the variance of the number of speeding tickets issued on Route 19 is greater than the variance of the number of speeding tickets issued on Route 22.

7. $(\sigma_1 \leq \sigma_2) \land (\sigma_1 > \sigma_2)$ (claim); $C.V. = 1.47; \alpha = 0.10; d.f.N. = 64; d.f.D. = 41; F = 2.32; reject. There is enough evidence to support the claim that the variance in the number of days factory workers miss per year due to illness is greater than the variation in the number of days hospital workers miss per year.

9. $H_0: \sigma_1 = \sigma_2$ and $H_1: \sigma_1 > \sigma_2$ (claim); $C.V. = 2.05; \alpha = 0.10; d.f.N. = 9; d.f.D. = 9; F = 5.06. The $P$-value for the $F$ test is $0.01 < P-value < 0.025 (0.012); reject since $P-value < 0.05$. There is enough evidence to support the claim that it is warmer in Birmingham.

11. $H_0: \sigma_1 = \sigma_2$ and $H_1: \sigma_1 > \sigma_2$ (claim); $C.V. = 1.47; \alpha = 0.10; d.f.N. = 64; d.f.D. = 41; F = 2.32; reject. There is enough evidence to support the claim that the variance in the number of days hospital workers miss per year is different. A cafeteria manager would want to know the results to make a decision on which beverage to serve.

13. $H_0: \mu_0 = 0$ and $H_1: \mu_0 < 0$ (claim); $C.V. = 1.895; d.f. = 7; t = -2.73; reject. There is enough evidence to support the claim that the mean has increased.

15. $\hat{p}_1 = 0.15; \hat{p}_2 = 0.05; \bar{p} = 0.104; \bar{q} = 0.896; H_0: \hat{p}_1 = \hat{p}_2$ and $H_1: \hat{p}_1 \neq \hat{p}_2$ (claim); $C.V. = 1.96; z = 2.41; reject. There is enough evidence to support the claim that the proportion has changed. 0.023 < \hat{p}_1 - \hat{p}_2 < 0.177. The confidence level does not contain 0; hence, the null hypothesis is rejected.

**Chapter Quiz**

1. False
2. False
3. True
4. False
5. d
6. a
7. c
8. b